

Worldsheet one-loop energy correction to IIA Giant Magnon

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Abstract

We compute one-loop corrections to the energy of a IIA giant magnon solution in the $AdS_4 \times CP^3$ background by using the standard quantum field theory (QFT) techniques. The string action is expanded around the solution to the quadratic order in the fluctuation fields. The resulting action has 2D coordinate dependent-coefficients, a feature that complicates the analysis. The solution contains a worldsheet velocity parameter v , and is expanded in terms of the parameter. A perturbative analysis is carried out by treating the v -dependent parts as vertices. The energy is computed by first putting the system in a box of length L and Fourier-transforming the fields into the discrete momentum modes. We compare our result with the results obtained by the algebraic curve method.

1 Introduction

A giant magnon (GM) [1] is a relatively simple solution of type II string sigma model whose dispersion relation has been known with a certain precision. In this work we consider a giant magnon of IIA string theory in $AdS_4 \times CP^3$, and analyze the worldsheet one-loop energy shift by using the standard QFT fluctuation techniques.

The exact dispersion relation of a magnon in $AdS_4 \times CP^3$ is [2–5]

$$\Delta \equiv E - J = \sqrt{\frac{Q^2}{4} + 4h^2(\lambda) \sin^2 \frac{p}{2}} \quad (1)$$

where Q is the number of the magnons and λ is t'Hooft coupling. (p is a parameter that is related to the worldsheet velocity v by (20) below.) The function $h(\lambda)$ can be interpolated between strong and weak coupling limits, and has the strong coupling expansion of the form

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + c + \dots \quad (2)$$

We will examine the small v region of the GM solution with $Q = 1$ below. The magnon becomes gradually larger as the parameter v approaches the $v \rightarrow 0$ limit. The dispersion relation (1) can be expanded as¹

$$\Delta = \sqrt{2\lambda} \left(1 - \frac{1}{2}v^2\right) + 2c \left(1 - \frac{1}{2}v^2\right) + \mathcal{O}(v^4) \quad (3)$$

The first paper in which c was computed is [6], and $c = 0$ was obtained. Using a new summing method, the authors of [7] obtained $c = -\frac{\ln 2}{2\pi}$. Both of these works employed the algebraic curve method [8] [9]. Below we determine the constant c using the standard worldsheet fluctuation technique.

Although the IIA GM is a relatively simple solution, the worldsheet analysis becomes much more complicated compared with, e.g., the circular or folded string configurations [10–16]. This is due to the fact that the fluctuation lagrangian around a GM solution has 2D coordinate-dependent coefficients. This feature makes determination of the eigenvalues of the kinetic operator very nontrivial. Nevertheless, the one-loop energy shift can be computed, as we show in this paper, as a perturbative series in the parameter v .

¹Presumably the $-\sqrt{\frac{\lambda}{2}} v^2$ term should come from spacetime loop(s).

The rest of the paper is organized as follows. In the next section, we briefly review the GM solution of [1]. We start with the IIA nonlinear sigma model in a general IIA supergravity curved background. The quadratic action that results from expanding the starting action around the GM solution has 2D coordinate dependent coefficients. The one-loop correction to energy is computed as a series of the parameter v in section 3. We start with the bosonic sector. The leading v^0 order is computed in dimensional regularization. The fermionic sector requires more care. The sector involves kappa symmetry fixing. We observe that a commonly used gauge choice is at odds with the worldsheet Lorentz invariance, and the magnon solution suggests a different but natural fixing. The coefficient c turns out to be different from the algebraic results, and is given in (76). In the conclusion, we comment on the absence of v -linear order in accordance with (3). We end with summary and future directions.

2 Quadratic action around Giant Magnon

To set the stage for the next section where we conduct the one-loop analysis, we briefly review the GM solution of [1]. The required quadratic action can be obtained by expanding the $AdS_4 \times CP_3$ background action around the GM solution.

An important point is that the *supergravity* action must be in a consistent convention with the nonlinear sigma model *string* action. As widely known, the supergravity action can be obtained from the corresponding string action (e.g., IIA supergravity from IIA superstring) by imposing kappa symmetry on the string action in a general supergravity background. One must substitute the $AdS_4 \times CP^3$ solution of the resulting supergravity action back into the *original* string sigma model action. This way, the uniformity of the convention is ensured, which of course is required for the consistency of the analysis. Throughout the paper, we employ the conventions of [17] both for the string and supergravity action.²

²The work of [17] was in the context of the membrane and 11D supergravity. For our purpose, therefore, it is necessary to reduce those theories to 10D. The reduction to IIA string was carried out in [18]. (Several re-scalings were introduced therein in order to put the sigma model action into the standard form. Here we undo those re-scalings to remain within the conventions of [17], i.e., to assure the use of the IIA supergravity action that comes directly out of reduction 11D supergravity.)

2.1 review $AdS_4 \times \mathbb{CP}^3$

The supergravity action can be obtained by dimensional reduction of the membrane and 11D supergravity action given in [17]. IIA supergravity admits an $AdS_4 \times CP_3$ solution; it is given, in string frame, by

$$ds_{IIA}^2 = ds_{AdS_4}^2 + ds_{\mathbb{CP}^3}^2 \quad (4)$$

$$F_{mn} = k \partial_{[m} A_{n]} \quad (5)$$

$$F_{(4)} = \frac{3}{8} k R^2 \epsilon_{(4)} \quad (6)$$

$$e^\Phi = \frac{R}{k} \quad (7)$$

where $\epsilon_{(4)}$ is the Levi-Civita symbol on AdS_4 , R is the radius of the curvature, \mathbf{J} is the Kähler form on \mathbb{CP}^3 , and k is an integer-valued constant (that corresponds to the level of ABJM theory). The metrics for the AdS_4 part and \mathbb{CP}^3 part are given respectively by

$$ds_{AdS_4}^2 = \frac{R^2}{4} \left[-\cosh^2 \hat{\rho} dt^2 + d\hat{\rho}^2 + \sinh^2 \hat{\rho} (d\hat{\theta}^2 + \sin^2 \hat{\theta} d\hat{\phi}^2) \right] \quad (8)$$

and

$$ds_{\mathbb{CP}^3}^2 = R^2 \left[d\hat{\xi}^2 + \cos^2 \hat{\xi} \sin^2 \hat{\xi} \left(d\hat{\psi} + \frac{1}{2} \cos \hat{\theta}_1 d\hat{\varphi}_1 - \frac{1}{2} \cos \hat{\theta}_2 d\hat{\varphi}_2 \right)^2 \right. \\ \left. + \frac{1}{4} \cos^2 \hat{\xi} (d\hat{\theta}_1^2 + \sin^2 \hat{\theta}_1 d\hat{\varphi}_1^2) + \frac{1}{4} \sin^2 \hat{\xi} (d\hat{\theta}_2^2 + \sin^2 \hat{\theta}_2 d\hat{\varphi}_2^2) \right] \quad (9)$$

with

$$0 \leq \hat{\xi} < \frac{\pi}{2}, \quad 0 \leq \hat{\psi} < 2\pi, \quad 0 \leq \hat{\theta}_i \leq \pi, \quad 0 \leq \hat{\varphi}_i < 2\pi. \quad (10)$$

In this coordinate system, the Kähler form on \mathbb{CP}^3 is given by $\mathbf{J} = R^2 dA$ where

$$A = \frac{1}{2} (\cos \hat{\theta}_1 \cos^2 \hat{\xi} d\hat{\varphi}_1 + \cos \hat{\theta}_2 \sin^2 \hat{\xi} d\hat{\varphi}_2 + \cos 2\hat{\xi} d\hat{\psi}). \quad (11)$$

One finds the following bosonic part of the nonlinear sigma model lagrangian:

$$\mathcal{L}_B = \frac{R^2}{4} \left\{ -\cosh^2 \hat{\rho} (\partial \hat{t})^2 + (\partial \hat{\rho})^2 + \sinh^2 \hat{\rho} [(\partial \hat{\theta})^2 + \sin^2 \hat{\theta} (\partial \hat{\phi})^2] \right\} \\ + R^2 \left\{ (\partial \hat{\xi})^2 + \cos^2 \hat{\xi} \sin^2 \hat{\xi} \left(\partial \hat{\psi} + \frac{1}{2} \cos \hat{\theta}_1 \partial \hat{\varphi}_1 - \frac{1}{2} \cos \hat{\theta}_2 \partial \hat{\varphi}_2 \right)^2 \right. \\ \left. + \frac{1}{4} \cos^2 \hat{\xi} [(\partial \hat{\theta})_1^2 + \sin^2 \hat{\theta}_1 (\partial \hat{\varphi}_1)^2] + \frac{1}{4} \sin^2 \hat{\xi} [(\partial \hat{\theta})_2^2 + \sin^2 \hat{\theta}_2 (\partial \hat{\varphi}_2)^2] \right\} \quad (12)$$

Since $\hat{\rho} = 0$ in the global coordinate system is degenerate, it is useful to use Cartesian coordinates, in which the metric takes

$$ds_{AdS_4}^2 = \frac{R^2}{4} \left[- \left(\frac{1 + \hat{\eta}^2}{1 - \hat{\eta}^2} \right)^2 d\hat{t}^2 + \frac{4}{(1 - \hat{\eta}^2)^2} d\vec{\hat{\eta}} \cdot d\vec{\hat{\eta}} \right] \quad (13)$$

These coordinates are related to the global coordinates by

$$\cosh \hat{\rho} = \frac{1 + \hat{\eta}^2}{1 - \hat{\eta}^2} \quad (14)$$

and only valid for $\hat{\eta}^2 = \vec{\hat{\eta}} \cdot \vec{\hat{\eta}} = \hat{\eta}_1^2 + \hat{\eta}_2^2 + \hat{\eta}_3^2 < 1$.

2.2 $R \times S^2$ GM solution

The giant magnon solution that we consider has support in one diagonal S^2 sector of $AdS_4 \times \mathbb{CP}^3$, and is given by³

$$\hat{t} = \tau \quad \hat{\eta}_i = 0 \quad \hat{\xi} = \frac{\pi}{4} \quad \hat{\theta}_i = \theta_{0i} \quad \hat{\varphi}_i = \varphi_{0i} \quad \hat{\psi} = 0 \quad (16)$$

where

$$\begin{aligned} \theta_0(x) &= \cos^{-1} \left(\frac{1}{\gamma} \text{sech} x \right), \\ \varphi_0(x) &= \tau + \tan^{-1} \left(\frac{1}{\gamma v} \tanh x \right) \end{aligned} \quad (17)$$

and

$$x = \gamma(\sigma - v\tau) \quad , \quad \gamma^2 = \frac{1}{1 - v^2}, \quad (18)$$

The parameter v is the worldsheet velocity of the magnon. θ_{0i} and φ_{0i} have been set to θ_0 and φ_0 respectively:

$$\theta_{01} = \theta_{02} \equiv \theta_0 \quad \text{and} \quad \varphi_{01} = \varphi_{02} \equiv \varphi_0. \quad (19)$$

³The classical energy of the solution is given by

$$E = \frac{R^2}{4\pi\alpha'} 2 \int_0^{2\pi} d\sigma \cosh^2 \rho \quad \dot{t} = \sqrt{2\lambda} \quad (15)$$

which is the leading piece of (3).

The momentum p of the magnon is related to v according to

$$v = \cos \frac{p}{2} \quad , \quad \gamma^{-1} = \sin \frac{p}{2} \quad (20)$$

In a generic coordinate system \hat{X}^M , the bosonic part of the Virasoro constraints are given by

$$G_{MN} \left(\dot{\hat{X}}^M \dot{\hat{X}}^N + \hat{X}'^M \hat{X}'^N \right) = 0 \quad (21)$$

$$G_{MN} \dot{\hat{X}}^M \hat{X}'^N = 0 \quad (22)$$

Since we will only consider the linear order of the Virasoro constraints, the fermionic part will not contribute. In terms of the angular coordinates introduced above, these translate to

$$0 = -\frac{1}{2}\dot{t} + \frac{1}{\sqrt{8}} \sin^2 \theta_0 (\dot{\varphi}_0 \dot{\varphi}_+ + \varphi'_0 \varphi'_+) + \frac{1}{\sqrt{8}} (\dot{\theta}_0 \dot{\theta}_+ + \theta'_0 \theta'_+) + \frac{1}{2\sqrt{8}} \sin 2\theta_0 (\dot{\varphi}_0^2 + \varphi'^2_0) \theta_+ \quad (23)$$

and

$$0 = -\frac{1}{2}\dot{t}' + \frac{1}{\sqrt{8}} \sin^2 \theta_0 (\dot{\varphi}_0 \varphi'_+ + \varphi'_0 \dot{\varphi}_+) + \frac{1}{\sqrt{8}} (\dot{\theta}_0 \theta'_+ + \theta'_0 \dot{\theta}_+) + \frac{1}{\sqrt{8}} \sin 2\theta_0 \dot{\varphi}_0 \varphi'_0 \theta_+ \quad (24)$$

at the linear order in the fields. (The constraints at the zeroth order in the fields are automatically satisfied.) Above, we have introduced

$$\begin{aligned} \varphi_+ &\equiv \frac{1}{\sqrt{2}} (\varphi_1 + \varphi_2) \\ \theta_+ &\equiv \frac{1}{\sqrt{2}} (\theta_1 + \theta_2) \end{aligned} \quad (25)$$

for convenience. The fields without " ^ " represent the fluctuation fields. Similarly, let us define

$$\begin{aligned} \varphi_- &\equiv \frac{1}{\sqrt{2}} (\varphi_1 - \varphi_2) \\ \theta_- &\equiv \frac{1}{\sqrt{2}} (\theta_1 - \theta_2) \end{aligned} \quad (26)$$

for later use. Note that the constraints are among t, φ_+, θ_+ at this order. (All the other fluctuation fields appear at the quadratic (and higher)-order expressions for the constraints.) This, with the structure of the quadratic lagrangian, naturally divides the bosonic sector into three sub-sectors as we discuss in section 3.

2.3 quadratic action

Upon substituting the solution into the sigma model action and expanding the resulting action, one can show, after lengthy algebra involving re-scalings, that

$$\begin{aligned}
-4\mathcal{L}_B^{(2)} = & (\partial t)^2 - 4 \sum_{i=1}^3 [(\partial\eta_i)^2 + \eta_i^2] - 8(\partial\psi)^2 - 8(\partial\xi)^2 \\
& - \frac{1}{2} \sin^2 \theta_0 (\partial\varphi_+)^2 - \frac{1}{2} (\partial\theta_+)^2 \\
& - \frac{1}{2} (1 - 2 \sin^2 \theta_0) (\partial\varphi_0)^2 \theta_+^2 - \sin 2\theta_0 \partial^a \varphi_0 \partial_a \varphi_+ \theta_+ \\
& - \frac{1}{2} (\partial\varphi_-)^2 - \frac{1}{2} (\partial\theta_-)^2 - \frac{1}{2} \cos^2 \theta_0 (\partial\varphi_0)^2 \theta_-^2 \\
& + 4 \left[\partial^a \theta_0 \partial_a \theta_- + \sin^2 \theta_0 \partial^a \varphi_0 \partial_a \varphi_- + \frac{1}{2} \sin 2\theta_0 (\partial\varphi_0)^2 \theta_- \right] \xi \\
& - \frac{1}{2} \sin 2\theta_0 [\partial^a \varphi_0 \partial_a \varphi_- \theta_-] \\
& + 4 \sin \theta_0 \partial^a \varphi_0 \partial_a \psi \theta_- - 4 \cos \theta_0 \partial^a \varphi_- \partial_a \psi.
\end{aligned} \tag{27}$$

for the bosonic part, and

$$-4\mathcal{L}_F = 4e^{\Phi/3} \bar{\Theta} (\eta^{ab} + \epsilon^{ab} \Gamma_{11}) e_a \left[\left(\partial_b + \frac{1}{4} w_b \right) + \Gamma \cdot F e_b \right] \Theta \tag{28}$$

for the fermionic part. Above,

$$\bar{\Theta} \equiv \Theta^\dagger \Gamma^0, \quad \epsilon^{\tau\sigma} = 1, \quad e_a \equiv \partial_a X^M e_M{}^A \Gamma_A, \quad w_a \equiv \partial_a X^M w_M{}^{AB} \Gamma_{AB} \tag{29}$$

and

$$\Gamma \cdot F \equiv \frac{1}{8} e^\phi (-\Gamma_{11} \Gamma \cdot F_2 + \Gamma \cdot F_4) \equiv \frac{1}{8} e^\phi \left[-\frac{1}{2} \Gamma_{11} \Gamma^{AB} (F_2)_{AB} + \frac{1}{4!} \Gamma^{ABCD} (F_4)_{ABCD} \right] \tag{30}$$

Since the fermionic action is already quadratic in Θ , it is only necessary to keep the leading-order (i.e., zeroth order) terms when evaluating the expressions above such as (30). Substituting the solution, one can show, in particular, that

$$\begin{aligned}
\Gamma \cdot F_2 &= \frac{2k}{R^2} (\Gamma^{45} + \Gamma^{67} + \Gamma^{89}) \\
\Gamma \cdot F_4 &= \frac{6k}{R^2} \Gamma^{0123} \quad \text{with } \varepsilon_{0123} = 1,
\end{aligned} \tag{31}$$

These results will be used in the analysis in the next section.

3 One-loop energy correction in v -series

Let us compute the one-loop energy shift for the magnon.⁴ Although we are dealing with the path integral that is quadratic in the fields, the coordinate dependence of the coefficients and nontrivial couplings between the fields make the full evaluation of the one-loop energy nontrivial. The analysis becomes more manageable once the lagrangian is expanded in terms of the small v , the velocity parameter that appears in the magnon solution.

Let us start with generalities, and consider

$$\int e^{i \int \mathcal{L}_0 + v \mathcal{L}_1 + v^2 \mathcal{L}_2 + \dots} \quad (32)$$

where \mathcal{L}_0 is the v -independent part; it is non-diagonal, and has coefficients that are functions of 2D space coordinates. \mathcal{L}_1 (\mathcal{L}_2) are the terms in linear (quadratic) order in v . Since the kinetic operator associated with \mathcal{L}_0 has position-dependent coefficients and the fields are coupled in nontrivial ways, explicit determination of the propagator still does not seem straightforward. For this reason, we first put the system in a 2D box of length L and go to the discrete 2D momentum space. After evaluating the one-loop energy, we convert the resulting expressions into continuous momentum space by taking the appropriate continuum limit.

Let us put the system in a 2D box of length L for each side, and consider the expansion of fields in terms of the following complete set, $\{e^{i\vec{p}_n \cdot \vec{z}}\}$, with

$$\vec{p}_n = (p_{n\tau}, p_{n\sigma}) = \frac{2\pi}{L} \vec{n} = \frac{2\pi}{L} (n_\tau, n_\sigma), \quad n_\sigma, n_\tau = 0, \pm 1, \dots, \pm \infty \quad (33)$$

The set satisfies the usual orthornormality condition

$$\int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{d^2 z}{L^2} e^{-i(\vec{p}_m - \vec{p}_n) \cdot \vec{z}} = \delta_{m-n, 0} \quad (34)$$

The actual analysis in section 3.1 reveals that the Fourier-transformed kinetic parts are diagonal after the $L \rightarrow \infty$ limit is taken. Let us consider the bosonic sector, and collectively denote the bosonic fields by Φ :

$$\Phi : \text{a collective representation for the bosonic fields} \quad (35)$$

⁴The $\text{AdS}_5 \times \text{S}_5$ case was considered in [19]. Some of the related works in $\text{AdS}_4 \times \text{CP}_3$ include [20–27].

The analysis for the fermionic sector is the same except for the usual sign change. Let us introduce the Fourier expansion

$$\Phi = \frac{1}{L} \sum_{\vec{n}} e^{i\vec{p}_n \cdot \vec{z}} \tilde{\Phi}_{\vec{n}} \quad \text{with} \quad \tilde{\Phi}_{\vec{n}}^\dagger = \tilde{\Phi}_{-\vec{n}} \quad (36)$$

where \vec{z} denotes the 2D worldsheet coordinates (τ, σ) ,

$$\vec{z} = (\tau, \sigma) \quad (37)$$

and the second equation originates from the reality of Φ . After adding the source terms, the v^0 -order action in the momentum space takes the following schematic form (the summation convention is understood)

$$-\tilde{\Phi}_{\vec{m}} M_{\vec{m}\vec{n}} \tilde{\Phi}_{\vec{n}} + \tilde{J}_{\vec{n}}^\dagger \tilde{\Phi}_{\vec{n}} + \tilde{\Phi}_{\vec{n}}^\dagger \tilde{J}_{\vec{n}} \quad (38)$$

where M_{mn}, J_n are the Fourier transformations of kinetic operator and the source term. The one-loop energy at v^0 -order comes from M_{mn} . Although the focus of this work is the v^0 -order, we present the general expression for the v -order energy for future purpose. The vertex terms (i.e., v -dependent terms) $\int \Phi V_{\Phi\Phi} \Phi$ go as

$$\int \frac{d^2 z}{L^2} \Phi V_{\Phi\Phi} \Phi = \frac{1}{L^2} \sum_{\vec{n}, \vec{m}} \int \frac{d^2 z}{L^2} \tilde{\Phi}_{\vec{m}}^\dagger e^{-i\vec{p}_m \cdot \vec{z}} V_{\Phi\Phi} \tilde{\Phi}_{\vec{n}} e^{i\vec{p}_n \cdot \vec{z}} \equiv \tilde{\Phi}_{\vec{m}}^\dagger (\tilde{V}_{\Phi\Phi})_{\vec{m}\vec{n}} \tilde{\Phi}_{\vec{n}} \quad (39)$$

The matrix $(\tilde{V}_{\Phi\Phi})_{\vec{m}\vec{n}}$ is the Fourier transformation of $\tilde{V}_{\Phi\Phi}$,

$$(\tilde{V}_{\Phi\Phi})_{\vec{m}\vec{n}} \equiv \int \frac{d^2 z}{L^2} e^{-i\vec{p}_m \cdot \vec{z}} V_{\Phi\Phi} e^{i\vec{p}_n \cdot \vec{z}} \quad (40)$$

At the v -order, this leads to the following schematic expression for the one-loop energy:

$$(\tilde{V}_{\Phi\Phi})_{\vec{m}\vec{n}} M_{\vec{n}\vec{m}}^{-1} \quad (41)$$

where we keep the linear order terms in $V_{\Phi\Phi}$. The inverse in M^{-1} should be taken in the tensor product space of the 4 by 4 matrix space and the space of the (m, n) indices. This would be a highly memory-demanding procedure in Mathematic computation. Fortunately, however, the matrix M turns out to be diagonal in the (m, n) space in the large L limit, and the inverse-taking procedure becomes simple.

3.1 bosonic part

We compute the one-loop energy by adopting conformal gauge (which we discuss in sec 3.1.3). The bosonic part can be divided into three sectors, the η sector, $(\psi, \xi, \varphi_-, \theta_-)$ sector and (t, θ_+, φ_+) sector. The Viroso constraints pertain to (t, θ_+, φ_+) sector.

3.1.1 η sector

The η sector of the lagrangian takes a simple form with constant coefficients, and one can simply adopt the usual approach of computing the one-loop energy. However, we take this sector to illustrate the procedure that we will use heavily in the other sectors, and demonstrate that the procedure reproduces the standard result when applied to a case with constant coefficients. While doing so, we find proper normalizations as well.

Putting the system in a box with area L^2

$$\begin{aligned}
\int (-4) \mathcal{L}_\eta^{(2)} &\Rightarrow (-4R^2) \int [(\partial \eta_i)^2 + \eta_i^2] \\
&= (-4R^2) \sum_{i=1}^3 \frac{1}{\sqrt{L^2}} \sum_{\vec{m}} \frac{1}{\sqrt{L^2}} \sum_{\vec{n}} \int \frac{d^2 z}{L^2} e^{i(\vec{p}_n - \vec{p}_m) \cdot \vec{z}} \tilde{\eta}_i^{\vec{m}\dagger} (\vec{p}_n^2 + 1) \tilde{\eta}_i^{\vec{n}} \\
&= (-4R^2) \frac{1}{L^2} \sum_{i=1}^3 \sum_{\vec{n}} \tilde{\eta}_i^{\vec{n}\dagger} (\vec{p}_n^2 + 1) \tilde{\eta}_i^{\vec{n}}
\end{aligned} \tag{42}$$

where we have used the proper normalization in which the sums over m, n come with $\frac{1}{\sqrt{L^2}}$. The path integral over $d\eta_i^n$ measure yields the following contribution

$$- \sum_{\vec{n}} \ln(\vec{p}_n^2 + 1) \tag{43}$$

for the one-loop energy. (The overall coefficient is irrelevant, not being recorded accurately.) After taking the continuum limit according to

$$\vec{p}_n \rightarrow \vec{p} \quad , \quad \frac{1}{L^2} \sum_{\vec{n}} \rightarrow \int d^2 p \tag{44}$$

one obtains the standard expression

$$-3 \int d^2 p \ln(\vec{p}^2 + 1) \tag{45}$$

where the factor 3 came from \sum_i^3 .

3.1.2 $(\psi, \xi, \varphi_-, \theta_-)$ sector

The lagrangian for this sector is given by

$$\begin{aligned}
[-4\mathcal{L}_{(\psi, \xi, \varphi_-, \theta_-)}^{(2)}] = & -8(\partial\psi)^2 - 8(\partial\xi)^2 \\
& -\frac{1}{2}(\partial\varphi_-)^2 - \frac{1}{2}(\partial\theta_-)^2 - \frac{1}{2}\cos^2\theta_0(\partial\varphi_0)^2\theta_-^2 \\
& +4\left[\partial^a\theta_0\partial_a\theta_- + \sin^2\theta_0\partial^a\varphi_0\partial_a\varphi_- + \frac{1}{2}\sin 2\theta_0(\partial\varphi_0)^2\theta_-\right]\xi \\
& -\frac{1}{2}\sin 2\theta_0[\partial^a\varphi_0\partial_a\varphi_-\theta_-] \\
& +4\sin\theta_0\partial^a\varphi_0\partial_a\psi\theta_- - 4\cos\theta_0\partial^a\varphi_-\partial_a\psi. \tag{46}
\end{aligned}$$

In terms of the Fourier modes, the action can be rewritten as

$$\frac{1}{L^2} \sum_{\vec{n}, \vec{m}} \int \frac{d^2z}{L^2} e^{i(p_n - p_m) \cdot z} (\tilde{\psi}^{m\dagger} \tilde{\xi}^{m\dagger} \tilde{\varphi}_-^{m\dagger} \tilde{\theta}_-^{m\dagger}) M_{4 \times 4} \left(\tilde{\psi}^n \tilde{\xi}^n \tilde{\varphi}_-^n \tilde{\theta}_-^n \right)^T \tag{47}$$

where $M_{4 \times 4}$ is a 4 by 4 matrix given by

$$\begin{aligned}
& -8p_n^2, \quad 0, \quad -2\cos\theta_0 p_m^a p_{an}, \quad -2i\sin\theta_0 \partial_a \varphi_0 p_n^a - \partial_a(\sin\theta_0 \partial^a \varphi_0) \\
& 0, \quad -8p_n^2, \quad 2i\sin^2\theta_0 \partial_a \varphi_0 p_m^a - \partial^a(\sin^2\theta_0 \partial_a \varphi_0), \quad 2i\partial_a \theta_0 p_m^a - \partial^a \partial_a \theta_0 + \sin 2\theta_0 (\partial\varphi_0)^2 \\
& -2\cos\theta_0 p_m^a p_{an}, \quad -2i\sin^2\theta_0 \partial_a \varphi_0 p_m^a - \partial^a(\sin^2\theta_0 \partial_a \varphi_0), \quad -\frac{1}{2}p_n^2, \quad \frac{ip_m^a}{4} \sin 2\theta_0 (\partial_a \varphi_0) + \frac{1}{8} \partial_a(\sin 2\theta_0 \partial^a \varphi_0) \\
& 2i\sin\theta_0 \partial_a \varphi_0 p_n^a - \partial_a(\sin\theta_0 \partial^a \varphi_0), \quad -2i\partial_a \theta_0 p_m^a - \partial^a \partial_a \theta_0 + \sin 2\theta_0 (\partial\varphi_0)^2, \quad -\frac{ip_m^a}{4} \sin 2\theta_0 (\partial_a \varphi_0) + \frac{1}{8} \partial_a(\sin 2\theta_0 \partial^a \varphi_0) \\
& , \quad -\frac{1}{2}(p_n^2 + \cos^2\theta_0 (\partial\varphi_0)^2) \tag{48}
\end{aligned}$$

We made heavy use of Mathematica in the following analysis. It is convenient to split the sum over (\vec{n}, \vec{m}) into four sectors. Defining $\vec{n} = (n_\tau, n_\sigma)$, $\vec{m} = (m_\tau, m_\sigma)$, they are

$$(n_\tau \neq m_\tau, n_\sigma \neq m_\sigma), \quad (n_\tau = m_\tau, n_\sigma = m_\sigma), \quad (n_\tau \neq m_\tau, n_\sigma = m_\sigma), \quad (n_\tau = m_\tau, n_\sigma \neq m_\sigma) \tag{49}$$

At the v^0 -order, the nonzero contributions come from $(n_\tau = m_\tau, n_\sigma = m_\sigma)$ and $(n_\tau = m_\tau, n_\sigma \neq m_\sigma)$ sectors. After taking $L \rightarrow \infty$ limit, the $(n_\tau = m_\tau, n_\sigma \neq m_\sigma)$ sector, Mathematica produces an expression that is antisymmetric in the tensor product space of the 4 by 4 matrix and the (\vec{n}, \vec{m}) space, which therefore, does not contribute to the one-loop correction.

For the $(n_\tau = m_\tau, n_\sigma = m_\sigma)$ sector, one finds

$$R^2 \text{diag}(-8\vec{p}_n^2, -8\vec{p}_n^2, -\frac{1}{2}\vec{p}_n^2, -\frac{1}{2}\vec{p}_n^2) \quad (50)$$

As a matter of fact, some of the off-diagonal entries survive after Fourier transformation; they are irrelevant for the one-loop energy. This is because the off-diagonal part is antisymmetric and therefore is removed by taking trace in the one-loop computation. Let us illustrate how all of the off-diagonal entries vanish with the (1,3) entry that contains $\cos \theta_0$ in the $(n_\tau = m_\tau, n_\sigma = m_\sigma)$ sector. The integral produces

$$\frac{4}{L} \arctan \left(\tanh \left(\frac{L}{4} \right) \right) \quad (51)$$

One can show that this expression vanishes in the large- L limit. Performing the path integral over $(\tilde{\psi}^n \tilde{\xi}^n \tilde{\varphi}_-^n \tilde{\theta}_-^n)$ and their conjugates and going to the continuum limit, the relevant part of the one-loop energy contribution from (50) is given by

$$-4 \int d^2 p \ln p^2 \quad (52)$$

3.1.3 (t, θ_+, φ_+) sector

The (t, θ_+, φ_+) sector of the bosonic action is

$$\begin{aligned} [-4\mathcal{L}_B]_{t, \theta_+, \varphi_+} &= (\partial t)^2 - \frac{1}{2}(\partial \theta_+)^2 - \frac{1}{2}(1 - 2 \sin^2 \theta_0)(\partial \varphi_0)^2 \theta_+^2 \\ &\quad - \sin 2\theta_0 \partial^a \varphi_0 \partial_a \varphi_+ \theta_+ - \frac{1}{2} \sin^2 \theta_0 (\partial \varphi_+)^2 \end{aligned} \quad (53)$$

For this sector, the Virosoro constraints (23) and (24) must be taken into account. Using the residual symmetry, one can impose

$$t = 0 \quad (54)$$

The action (53) then becomes

$$\begin{aligned}
[-4\mathcal{L}_B]_{\theta_+, \varphi_+} &= -\frac{1}{2}(\partial\theta_+)^2 - \frac{1}{2}(1 - 2\sin^2\theta_0)(\partial\varphi_0)^2\theta_+^2 \\
&\quad - \sin 2\theta_0\partial^a\varphi_0\partial_a\varphi_+\theta_+ - \frac{1}{2}\sin^2\theta_0(\partial\varphi_+)^2
\end{aligned} \tag{55}$$

The Virosoro constraints now take

$$\begin{aligned}
\sin^2\theta_0(\dot{\varphi}_0\dot{\varphi}_+ + \varphi'_0\varphi'_+) + (\dot{\theta}_0\dot{\theta}_+ + \theta'_0\theta'_+) + \frac{1}{2}\sin 2\theta_0(\dot{\varphi}_0^2 + \varphi'^2_0)\theta_+ &= 0 \\
\sin^2\theta_0(\dot{\varphi}_0\varphi'_+ + \varphi'_0\dot{\varphi}_+) + (\dot{\theta}_0\theta'_+ + \theta'_0\dot{\theta}_+) + \sin 2\theta_0\dot{\varphi}_0\varphi'_0\theta_+ &= 0
\end{aligned} \tag{56}$$

These can easily be solved for $\dot{\varphi}_+$, φ'_+ , and the solutions can be substituted into (55). The rest of the analysis is similar to that of the previous section; the one-loop contribution from this sector is

$$-\int d^2p \ln(p^2 + 1) \tag{57}$$

3.2 fermionic part

Let us quote the quadratic fermionic action for convenience:

$$-4\mathcal{L}_F = 4e^{\Phi/3}\bar{\Theta}(\eta^{ab} + \epsilon^{ab}\Gamma_{11})e_a \left[\left(\partial_b + \frac{1}{4}w_b \right) + \Gamma \cdot F e_b \right] \Theta \tag{58}$$

The fermionic coordinates are such that $\Theta = \Theta^1 + \Theta^2$ with

$$\Theta^1 \equiv \begin{pmatrix} \Theta^1 \\ 0 \end{pmatrix}, \quad \Theta^2 \equiv \begin{pmatrix} 0 \\ \Theta^2 \end{pmatrix} \tag{59}$$

The matrix that appears in (58) turns out to be half-ranked when the magnon solution is substituted. In other words, half of the components in Θ do not appear when the action (58) is expanded. This naturally suggest a gauge fixing in which one set those components to zero.⁵ Consider the following Fourier transformation

$$\Theta = \sum_n e^{i\vec{p}_n \cdot \vec{z}} \tilde{\Theta}_n \tag{63}$$

⁵To our surprise, a certain kappa-symmetry gauge choice seems incompatible (at least apparently) with the worldsheet Lorentz invariance. We illustrate this point with an innocuous-looking gauge choice $(1 - \Gamma^{11})\Theta = 0$. It is easier to see the issue in flat space.

In the Fourier transformed space, one can separated the momentum-dependent part of the matrix in (58) from the momentum-independent part. It turns out that the momentum independent part vanishes by symmetric sum.

One subtle issue concerns how to take hermitian conjugation of the kinetic matrix in the \vec{n} -vector space. It turns out that the correct hermition conjugation in the momentum space involves $\vec{n} \rightarrow -\vec{n}$ as well.⁶ To see this in more detail, let us consider one of the momentum independent entries,

$$-\frac{R}{16} \sum_{-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} \Theta_{\vec{n}}^4 \quad (64)$$

where 1 and 4 are the spinor indices, and impose the hermiticity requirement using the the usual hermitian conjugation (, i.e., the one that does not involve $\vec{n} \rightarrow -\vec{n}$,)

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} \Theta_{\vec{n}}^4 &= \left(\sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} \Theta_{\vec{n}}^4 \right)^\dagger = \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{4\dagger} \Theta_{\vec{n}}^1 \\ &= - \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^1 \Theta_{\vec{n}}^{4*} = - \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^1 \Theta_{-\vec{n}}^4 = - \sum_{n=-\infty}^{\infty} \Theta_{-\vec{n}}^{1\dagger} \Theta_{-\vec{n}}^4 \\ &= - \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} \Theta_{\vec{n}}^4 \end{aligned} \quad (65)$$

A convenient choice for Γ^{11} is

$$\Gamma^{11} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (60)$$

The Θ^1, Θ^2 field equations take

$$(-\partial_\tau + \partial_\sigma) \Theta^1 = 0 \quad , \quad (\partial_\tau + \partial_\sigma) \Theta^2 = 0 \quad (61)$$

The particular gauge choice removes the entire Θ^2 (i.e., the left-moving modes) thereby breaking the 2D Lorentz invariance of the fermionic part of the Virasoro constraints (23) (24) that have been suppressed. Although it might be possible to restore the 2D invariance at the end, it might take some complicated steps. If κ -fixing were necessary, the following gauge fixing would be a good choice for example:

$$(\Gamma^0 + \Gamma^1) \Theta = 0 \quad (62)$$

⁶This subtlety should be attributed to the fact that both positive and negative integers were used to label the matrix in the \vec{n} -vector space.

Therefore, the usual hermitian conjugation, lead to

$$\sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} \Theta_{\vec{n}}^4 = 0 \quad (66)$$

As a matter of fact even if one uses the hermitian conjugation that *does* involve $\vec{n} \rightarrow -\vec{n}$, one gets the same result. However, those two conjugations lead to different results for momentum dependent terms to which we now turn; for example consider

$$\frac{iR}{2\sqrt{2}} \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^9 \quad (67)$$

The usual hermitian conjugation yields

$$\begin{aligned} & i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^9 = \left(i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^9 \right)^\dagger \\ &= -i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{9\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^1 = i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^1 (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^{9*} \\ &= i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^1 (p_{n_\tau} + p_{n_\sigma}) \Theta_{-\vec{n}}^9 = i \sum_{n=-\infty}^{\infty} \Theta_{-\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{-\vec{n}}^9 \\ &= -i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^9 \end{aligned} \quad (68)$$

Therefore one gets

$$i \sum_{n=-\infty}^{\infty} \Theta_{\vec{n}}^{1\dagger} (p_{n_\tau} + p_{n_\sigma}) \Theta_{\vec{n}}^9 = 0 \quad (69)$$

Because of this, the entire fermionic matrix vanishes, and this cannot be true. If one uses the hermitian conjugation that does involve $\vec{n} \rightarrow -\vec{n}$, additional minus sign appears in the far right hand side of (68), making the result non-vanishing.

At the v^0 -order, the relevant part of one-loop contribution turns out to be

$$8 \int d^2p \ln p^2 \quad (70)$$

3.3 Combining bosonic and fermionic contributions

Let us combine the bosonic and fermionic results. The total energy v^0 order is given by

$$\begin{aligned} & - \int d^2p \left(4 \ln(p^2 + 1) + 4 \ln p^2 - 8 \ln p^2 \right) \\ = & - \int d^2p \left(4 \ln(p^2 + 1) - 4 \ln p^2 \right) \end{aligned} \quad (71)$$

The first term in the first line is a combination of the results from the η -sector and θ_+ -sector while the second term comes from the 4 by 4 sector. The third term comes from the fermionic sector. The $\int d^2p 4 \ln(p^2 + 1)$ can be evaluated as follows. Let us define

$$\delta E_{(\eta, \theta_+)} = -4 \int d^2p \ln(\vec{p}^2 + m^2) \quad (72)$$

where we have introduced a “mass” parameter, $m = 1$. In the dimensional regularization that we have adopted, this can be evaluated as follows:

$$\frac{\partial}{\partial m^2} \delta E_{(\eta, \theta_+)} = -4 \int d^2p \frac{1}{\vec{p}^2 + m^2} = -4 \frac{\Gamma(0)}{4\pi} \quad (73)$$

This implies

$$\delta E_{(\eta, \theta_+)} = -4 \frac{\Gamma(0)}{4\pi} m^2 + C = -\frac{\Gamma(0)}{\pi} + C \quad (74)$$

where we have used $m^2 = 1$ for the second equality. The constant C gets cancelled by the $\ln p^2$ term in (71):

$$\frac{\delta E}{\Gamma(0)} = -\frac{1}{\pi} \quad (75)$$

4 Conclusion

In this work, we have analyzed the worldsheet one-loop energy shift in dimensional regularization. Our analysis has led to the following value of the coefficient c that appears in (3),

$$c = -\frac{1}{\pi} \quad (76)$$

It will be worth checking whether the v^0 -order propagator could be determined analytically, and cross-checking various results obtained in this work. Presumably, clever field redefinitions would be required to that end. One obvious future direction is to compute the v^1 and v^2 -order energy shifts, and check the result against (3). As a matter of fact, we have carried out some preliminary calculations at the v order, and the result seems to indicate a vanishing outcome in accordance with (3). We plan to report on further clarification on v and v^2 orders in the near future. Finally, it would be useful to repeat computations by employing the formulations of [28] [29] [30]. These formulations have an advantage of the manifest $\text{AdS}_4 \times \text{CP}_3$ isometry.

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